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37. a) $2^x = 64$ → $x = 6$ $S = \{6\}$

$2^x = 2^6$

b) $3^{x-2} = 9$ → $x - 2 = 2$ $S = \{4\}$

$3^{x-2} = 3^2$ $x = 4$

c) $5^{x^2-2x} = 125$ → $x^2 - 2x = 3$ $\Delta = 16$ $S = \{-1, 3\}$

$5^{x^2-2x} = 5^3$ $x^2 - 2x - 3 = 0$ $x' = 3$

$x'' = -1$

d) $10^{1-x} = \frac{1}{10}$ → $1 - x = -1$ $S = \{2\}$

$10^{1-x} = 10^{-1}$ $x = 2$

e) $(\sqrt{2})^x = 4$ → $\frac{x}{2} = 2$ $S = \{4\}$

$2^{\frac{x}{2}} = 2^2$ $x = 4$

f) $(0,5)^{2x} = 2^{1-3x}$ → $-2x = 1 - 3x$ $S = \{1\}$

$\left(\frac{1}{2}\right)^{2x} = 2^{1-3x}$ $x = 1$

$2^{-2x} = 2^{1-3x}$

g) $2^{4x-x^2} = 8$ → $4x - x^2 = 3$ $\Delta = 4$ $S = \{1, 3\}$

$2^{4x-x^2} = 2^3$ $-x^2 + 4x - 3 = 0$ $x' = 1$

$x'' = 3$

h) $(10^x)^{1-x} = 0,000001$ → $-x^2 + x + 6 = 0$ $\Delta = 25$ $S = \{-2, 3\}$

$10^{-x^2+x} = 10^{-6}$ $x^2 - x - 6 = 0$ $x' = 3$

$x'' = -2$

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j) $3^{x-5} = 27^{1-x}$

$3^{x-5} = (3^3)^{1-x}$

$3^{x-5} = 3^{3-3x}$

$x-5 = 3-3x$

$4x = 8$

$x = 2$

$S = \{2\}$

i) $3^{2-x} = \frac{1}{27}$

$3^{2-x} = 3^{-3}$

$2-x = -3$

$x = 5$

$S = \{5\}$

k) $\left(\frac{1}{2}\right)^{x^2-4} = 8^{x+2}$

$2^{-x^2+4} = 2^{3x+6}$

$-x^2+4 = 3x+6$

$x^2+3x+2=0$

$\Delta = 9-4.1.2$

$\Delta = 1$

$x' = -1$

$x'' = -2$

$S = \{-2,-1\}$

l) $\sqrt[5]{2^x} = \frac{1}{32}$

$2^{\frac{x}{5}} = 2^{-5}$

$x/5 = -5$

$x = -25$

$S = \{-25\}$

38- $f(a) = g(a)$

$\left(\frac{4}{5}\right)^{4a^2-a} = \left(\frac{8}{10}\right)^{3a+3}$

$\left(\frac{4}{5}\right)^{4a^2-a} = \left(\frac{4}{5}\right)^{3a+3}$

$4a^2-4a-3=0$

$4a^2+2a-6a-3=0$

$2a(2a+1)-3(2a+1)=0$

$(2a+1)(2a-3)=0$

$2a+1=0$

$a = -\frac{1}{2}$

$2a-3=0$

$a = \frac{3}{2}$

39- $2.2^x = \sqrt{8}.\sqrt[4]{2}.\sqrt[6]{2}$

$2.2^x = 2^{\frac{3}{2}}.2^{\frac{1}{4}}.2^{\frac{1}{6}}$

$2.2^x = 2^{\frac{18+3+2}{12}}$

$2.2^x = 2^{\frac{23}{12}}$

$2^x = 2^{\frac{23-12}{12}}$

$2^x = 2^{\frac{11}{12}}$

$x = \frac{11}{12}$

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40- I $\begin{cases} 3^{x+y}=1 \\ 2^{x+2y}=2 \end{cases}$ II

(I) $3^{x+y}=1$

$3^{x+y}=3^0$

$x+y=0$

(II) $2^{x+2y}=2^1$

$x+2y=1$

$\begin{cases} x+y=0 \rightarrow x=-y \\ x+2y=1 \end{cases}$

$x+2y=1$

$-y+2y=1$

$y=1$

$x=-y$

$x=-1$

$x-y=?$

$-1-1=-2$

$\downarrow$

$x-y=-2$

41- I $\begin{cases} 4^x \cdot 8^y=\frac{1}{4} \\ 9^x \cdot 27^{2y}=3 \end{cases}$ II

(I) $2^{2x} \cdot 2^{3y}=2^{-2}$

$2^{2x+3y}=2^{-2}$

$2x+3y=-2$

(II) $9^x \cdot 27^{2y}=3$

$3^{2x} \cdot 3^{6y}=3^1$

$3^{2x+6y}=3^1$

$2x+6y=1$

$\begin{cases} 2x+3y=-2 \rightarrow 2x=-2-3y \\ 2x+6y=1 \end{cases}$

$2x+6y=1$

$-2-3y+6y=1$

$3y=3$

$y=1$

$2x=-2-3$

$2x=-5$

$x=\frac{-5}{2}$

$S=\{(-\frac{5}{2},1)\}$

42- I $\begin{cases} 5^{x-y}=\frac{1}{125} \\ 3^{x+y}=243 \end{cases}$ II

(I) $5^{x-y}=5^{-3}$

$x-y=-3$

(II) $3^{x+y}=243$

$3^{x+y}=3^5$

$x+y=5$

$\begin{cases} x-y=-3 \rightarrow x=y-3 \\ x+y=5 \end{cases}$

$x+y=5$

$y-3+y=5$

$2y=8$

$y=4$

$x=y-3$

$x=1$

$(x.y)^3=?$

$4^3=64$

$(x.y)^3=64$

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43- $4^{x-1} = 2^x$ $\quad 2x-2 = x$ $\quad f(x) = 4^{x-1}$ $\quad (2,4)$

$2^{2x-2} = 2^x$ $\quad x = 2$ $\quad f(x) = 4^1 = 4$

44- a) $2 \cdot 3^{x-2} = 162$ $\quad x-2 = 4$ $\quad S = \{6\}$

$3^{x-2} = 81$ $\quad x = 6$

$3^{x-2} = 3^4$

b) $3 \cdot 5^{x-1} = 75$ $\quad x-1 = 2$ $\quad S = \{3\}$

$\cancel{3} \cdot 5^{x-1} = \cancel{3} \cdot 25$ $\quad x = 3$

$5^{x-1} = 5^2$

c) $5 \cdot 2^{x^2-4} = 160$ $\quad x^2-4 = 5$ $\quad S = \{-3,3\}$

$\cancel{5} \cdot 2^{x^2-4} = \cancel{5} \cdot 32$ $\quad x = \pm\sqrt{9}$

$2^{x^2-4} = 2^5$ $\quad x = \pm 3$

d) $10 \cdot 2^{x+3} = 10$ $\quad x+3 = 0$ $\quad S = \{-3\}$

$2^{x+3} = \frac{10}{10}$ $\quad x = -3$

$2^{x+3} = 2^0$

45- a) $2^{x-3} + 2^{x-1} + 2^x = 52$ $\quad x = 5$

$2^x \cdot 2^{-3} + 2^x \cdot 2^{-1} + 2^x = 52$

$2^x(2^{-3} + 2^{-1} + 2^0) = 52$ $\quad S = \{5\}$

$2^x\left(\frac{1}{8} + \frac{1}{2} + 1\right) = 52$

$2^x\left(\frac{1+4+8}{8}\right) = 52$

$2^x\left(\frac{13}{8}\right) = 52$

$2^x = \frac{52 \cdot 8}{13}$

$2^x = 32$

$2^x = 2^5$

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b) $7^x + 7^{x-1} = 8$ $\quad x = 1 \quad S = \{1\}$

$7^x + 7^x \cdot 7^{-1} = 8$

$7^x(7^{-1} + 7^0) = 8$

$7^x\left(\frac{1}{7} + 1\right) = 8$

$7^x\left(\frac{8}{7}\right) = 8$

$7^x = \frac{8 \cdot 7}{8}$

$7^x = 7^1$

c) $4 \cdot 2^x + 2^{x-1} = 72$ $\quad x = 4 \quad S = \{4\}$

$4 \cdot 2^x + 2^x \cdot 2^{-1} = 2^3 \cdot 3^2$

$2^x(4 + 2^{-1}) = 2^3 \cdot 3^2$

$2^x\left(\frac{9}{2}\right) = 2^3 \cdot 3^2$

$2^x = \frac{2^3 \cdot 3^2 \cdot 2}{3^2}$

$2^x = 2^4$

d) $1 + \frac{3^x - 1}{3^x} = -1$

$\frac{3^x + 3^x - 1}{3^x} = -1$

Consideremos $y = 3^x$

$2 \cdot 3^x - 1 = -3^x$

$2y - 1 = -y$

$3y = 1$

$y = \frac{1}{3} \longrightarrow 3^x = \frac{1}{3}$ $\quad S = \{-1\}$

$3^x = 3^{-1}$

$x = -1$

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46-a) $3^{2x} + 2.3^{x} - 15 = 0$

$(3^{x})^{2} + 2.3^{x} - 15 = 0$

$y^{2} + 2y - 15 = 0$

Consideremos $y = 3^{x}$

$S = -2$ $\quad y' = 3 \longrightarrow y = 3^{x}$

$P = -15$ $\quad y'' = -5$ $\quad 3^{1} = 3^{x}$

$x = 1$

$\rightarrow y = 3^{x}$

$-5 = 3^{x}$

Em $\mathbb{R}$, nenhuma potência resulta em um número negativo

$\therefore$

$x'' \notin \mathbb{R}$

$\boxed{x = 1 \quad S = \{1\}}$

b) $2^{2x+1} + 3.2^{x+1} = 8$

$2^{2x}.2^{1} + 3.2^{x}.2^{1} = 8$

$(2^{x})^{2}.2^{1} + 3.2^{x}.2^{1} = 8$

$2y^{2} + 6y - 8 = 0$

$y^{2} + 3y - 4 = 0$

Consideremos $y = 2^{x}$

$S = -3$ $\quad y' = 1 \longrightarrow y' = 2^{x}$

$P = -4$ $\quad y'' = -4$ $\quad 2^{x} = 1$

$2^{x} = 2^{0}$

$x = 0$

$\boxed{S = \{0\}}$

$\rightarrow y'' = 2^{x}$

$2^{x} = -4$

$\downarrow$

$x \notin \mathbb{R}$

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c) $4^{x+2} - 3.2^{x+3} = 160$

$2^{2x+4} - 3.2^{x+3} = 160$

$(2^x)^2.2^4 - 3.2^x.2^3 = 160$

Consideremos $y = 2^x$

$16y^2 - 24y - 160 = 0$

$8y^2 - 12y - 80 = 0$

$4y^2 - 6y - 40 = 0$

$2y^2 - 3y - 20 = 0$

$\Delta = 9 - 4.2.(-20)$

$\Delta = 9 + 160$

$\Delta = 169$

$y' = 4 \longrightarrow y = 2^x$

$2^x = 4$

$2^x = 2^2$

$x = 2$

$y'' = -2,5$

$\rightarrow y = 2^x$

$2^x = -2,5$

$x \notin \mathbb{R}$

$\boxed{S = \{2\}}$

d) $\frac{9^x + 3}{4} - 3^x = 0$

$\frac{9^x + 3 - 4.3^x}{4} = 0$

$9^x + 3 - 4.3^x = 0$

$3^{2x} + 3 - 4.3^x = 0$

Consideremos $y = 3^x$

$y^2 - 4y + 3 = 0$

$S = 4$

$P = 3$

$y' = 1 \longrightarrow y = 3^x$

$3^x = 1$

$3^x = 3^0$

$x = 0$

$y'' = 3$

$y = 3^x$

$3^x = 3^1$

$x = 1$

$S = \{0, 1\}$

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e) $3^x - \frac{9}{3^x} = 8$

Consideremos $y = 3^x$

$y - \frac{9}{y} - 8 = 0$

$\frac{y^2 - 9 - 8y}{y} = 0$

$y^2 - 8y - 9 = 0$

$S = 8$ $\quad y' = 9 \longrightarrow y = 3^x$

$P = -9$ $\quad y'' = -1$ $\quad 3^x = 3^2$

$x = 2$

$\boxed{S = \{2\}}$

$\longrightarrow y = 3^x$

$3^x = -1$

$x \notin \mathbb{R}$

47- $3^x = 2187 \rightarrow 3^x = 3^7$

2187	3
729	3
243	3
81	3
27	3
9	3
3	3
1	

$\rightarrow$ A contagem começa em 3^0, logo 3^7 é o oitavo termo da sequência.