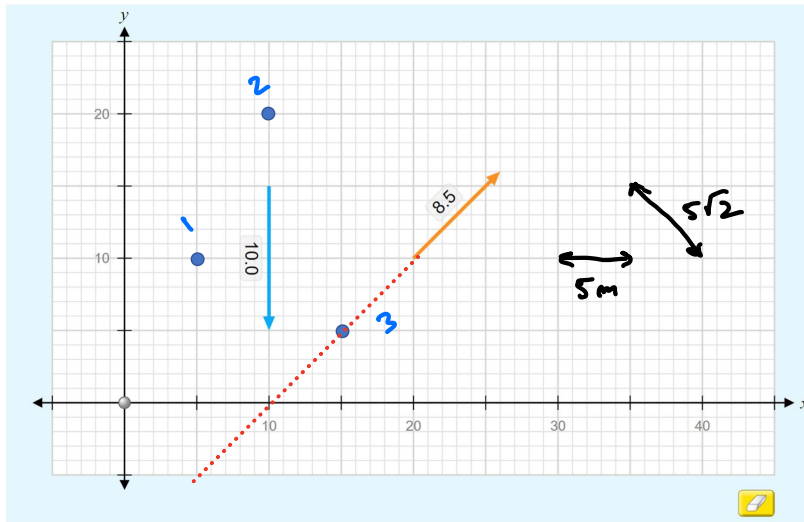


- Exercício: Calcule o torque produzido nos pontos a seguir pelas forças $F_A = 10\text{ N}$ e $F_B = 8,5\text{ N}$. Unidade do gráfico = 1 m .



Ponto 1

$$M_A = 10 \cdot 5 = 50 \Rightarrow -50 \text{ kNm}$$

$$M_B = 8,5 \cdot \left(5\sqrt{2} + \frac{5\sqrt{2}}{2} \right) =$$

$$M_B = 8,5 \cdot \frac{15\sqrt{2}}{2} = 90,2 \text{ kNm}$$

$$M_1 = -50 + 90,2$$

$$M_1 = 40,2 \text{ kNm}$$

Ponto 2

$$M_A = 10 \cdot 0 = 0$$

$$M_B = 8,5 \cdot 10\sqrt{2} = 120 \text{ kNm}$$

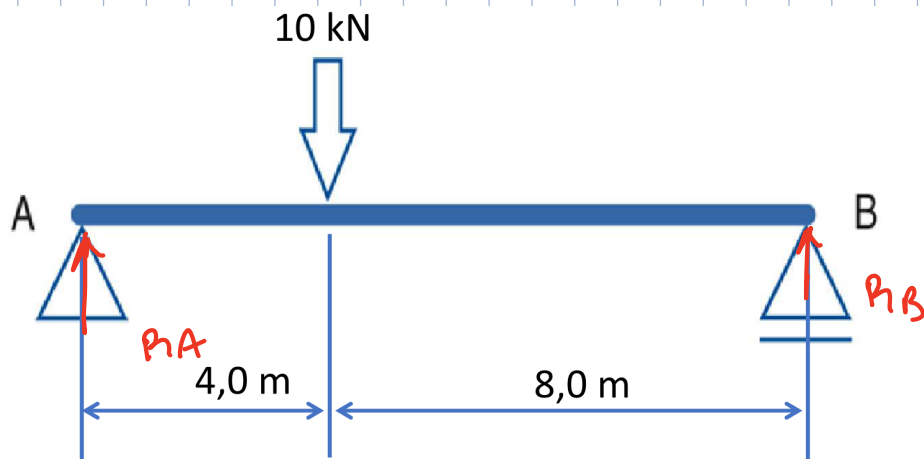
$$\left. \begin{array}{l} M_A = 0 \\ M_B = 120 \text{ kNm} \end{array} \right\} M_2 = 120 \text{ kNm}$$

Ponto 3

$$M_A = 10 \cdot 5 = 50$$

$$M_B = 8,5 \cdot 0 = 0$$

$$\left. \begin{array}{l} M_A = 50 \\ M_B = 0 \end{array} \right\} M_3 = 50 \text{ kNm}$$

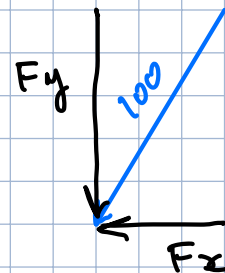
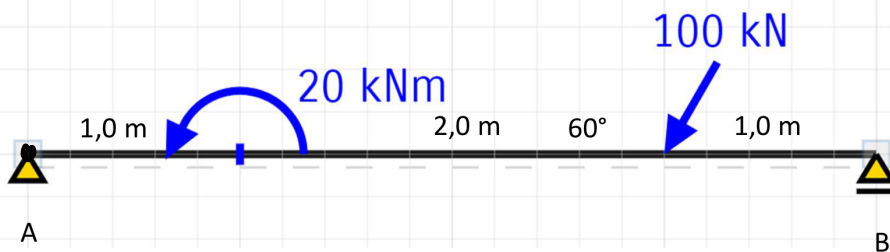


$$\sum F_v = 0 \Rightarrow R_A - 10 + R_B = 0 \Rightarrow R_A + R_B = 10$$

$$\sum F_H = 0 \Rightarrow \text{não há}$$

$$\sum M = 0 \quad R_A \cdot 0 - 10 \cdot 4 + R_B \cdot 12 \Rightarrow R_B = \frac{40}{12} \Rightarrow R_B = 3,3 \text{ kN}$$

$$R_A = 10 - 3,3 \Rightarrow R_A = 6,7 \text{ kN}$$



$$F_x = 100 \cdot \cos 60^\circ = 50 \text{ kN} \quad \vec{F}_x = -50 \text{ kN}$$

$$F_y = 100 \cdot \sin 60^\circ = 86,6 \text{ kN} \quad \vec{F}_y = -86,6 \text{ kN}$$

$$\sum F_v = 0 \Rightarrow R_{Ay} - 86,6 + R_{By} = 0 \Rightarrow R_{Ay} + R_{By} = 86,6$$

$$\sum F_H = 0 \Rightarrow R_{Ax} + (-50) = 0 \Rightarrow R_{Ax} = 50 \text{ kN} //$$

$$\sum M = 0 \Rightarrow R_{Ax} \cdot 0 + R_{Ay} \cdot 0 + 20 + F_x \cdot 0 - 86,6 \cdot 3 + R_{By} \cdot 4 = 0$$

$$20 - 260 + R_{By} \cdot 4 = 0$$

$$R_{By} = \frac{240}{4} \Rightarrow R_{By} = 60 \text{ kN} //$$

$$R_{Ay} = 86,6 - 60 \Rightarrow R_{Ay} = 27 \text{ kN} //$$

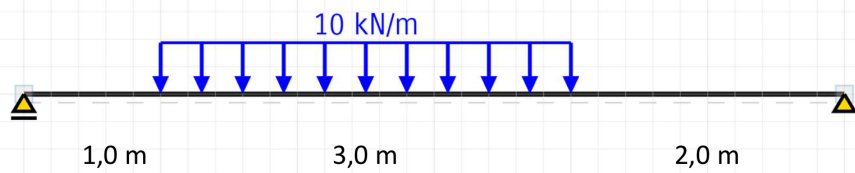
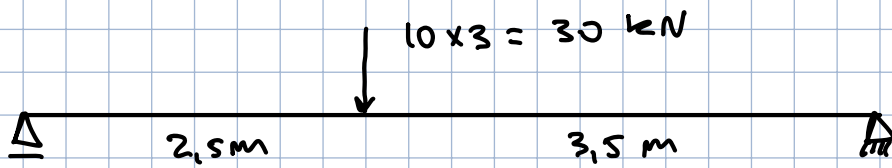


Diagrama equivalente

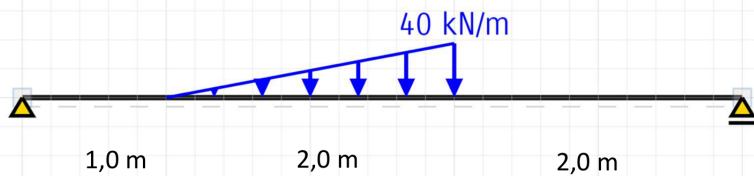


$$\sum F_v = 0 \Rightarrow R_A - 30 + R_B \Rightarrow R_A + R_B = 30$$

$$\sum F_H = 0 \Rightarrow \text{nao ha}$$

$$\sum M = 0 \Rightarrow R_A \cdot 0 - 30 \cdot 2.5 + R_B \cdot 6 \Rightarrow R_B = \frac{75}{6} \Rightarrow R_B = 12.5 \text{ kN}$$

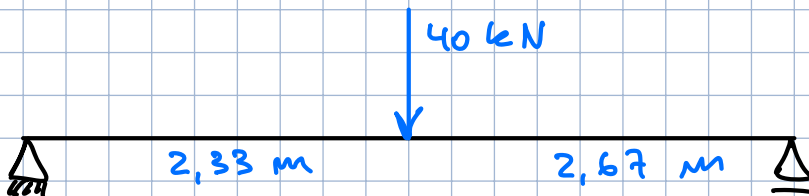
$$R_A = 30 - 12.5 \Rightarrow R_A = 17.5 \text{ kN}$$



$$F = \frac{2 \times 40}{2} = 40 \text{ kN}$$

$$\frac{2}{3} = 0.67$$

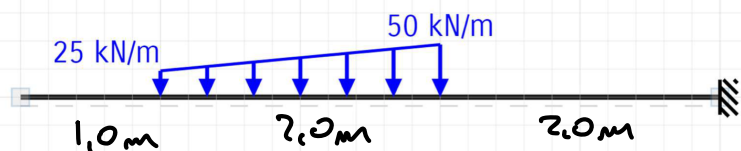
Diagrama equivalente



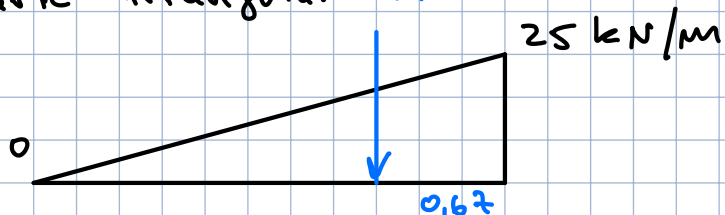
$$\sum F_v = 0 \Rightarrow R_A - 40 + R_B \Rightarrow R_A + R_B = 40$$

$$\sum M = 0 \Rightarrow R_A \cdot 0 - 40 \cdot 2.33 + R_B \cdot 5.0 = 0 \Rightarrow R_B = 18.6 \text{ N}$$

$$R_A = 40 - 18.6 \Rightarrow R_A = 21.4 \text{ N}$$



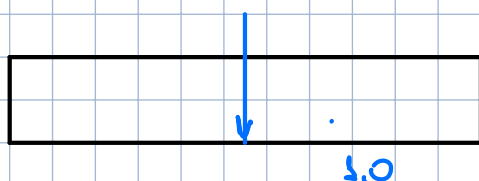
Parte triangular F_1



$$F_1 = \frac{2.0 \times 25}{2} = 25 \text{ kN}$$

$$\frac{2}{3} = 0.67 \text{ m}$$

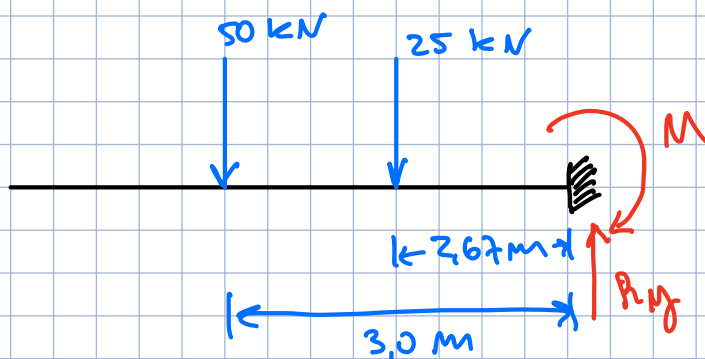
Parte rectangular



$$F_2 = 2.0 \cdot 20 = 50 \text{ N}$$

$$\frac{2}{2} = 1.0 \text{ m}$$

Diagrama equivalente



$$\sum F_v = 0$$

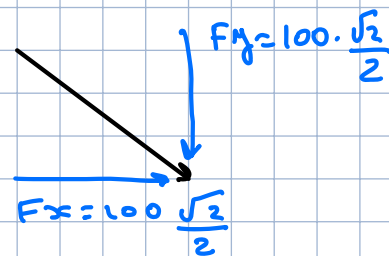
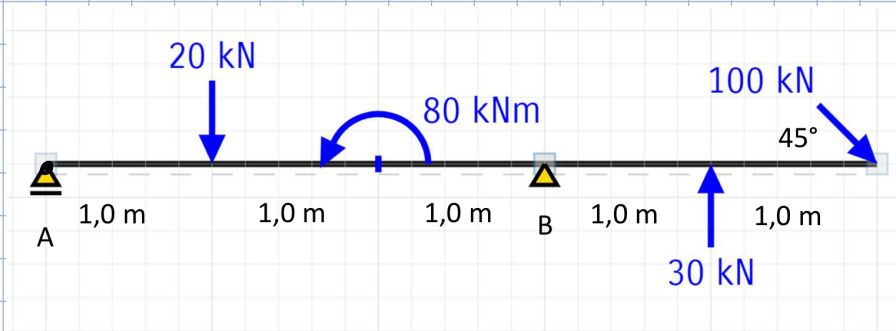
$$R_y - 50 - 25 = 0 \Rightarrow R_y = 75 \text{ kN}$$

$$\sum M = 0$$

$$50 \cdot 3 + 25 \cdot 2.67 - M = 0 \Rightarrow M = -217 \text{ kN.m}$$

Exercícios

①



$$\sum F_v = 0 \Rightarrow R_{A_y} - 20 + R_{B_y} + 30 - \frac{100\sqrt{2}}{2} = 0$$

$$R_{A_y} + R_{B_y} = -10 + 50\sqrt{2} = 60,7 \text{ kN}$$

$$\sum F_H = 0 \Rightarrow R_{B_x} + 50\sqrt{2} = 0 \quad R_{B_x} = -70,7 \text{ kN}$$

$$\sum M = 0 \quad R_A \cdot 0 - 20 \cdot 1 + 80 + R_{B_y} \cdot 3 + 30 \cdot 4 - 50\sqrt{2} \cdot 5 = 0$$

$$3R_{B_y} = -180 + 250\sqrt{2}$$

$$R_{B_y} = 57,8 \text{ kN}$$

$$R_{A_y} = 60,7 - 57,8 = 2,9 \text{ kN}$$

$$R_{A_y} = 2,9 \text{ kN} ; \quad R_{B_x} = 58 \text{ kN} ; \quad R_{B_y} = 71 \text{ kN}$$

2

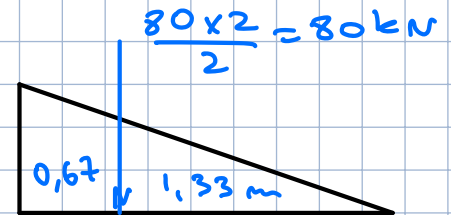
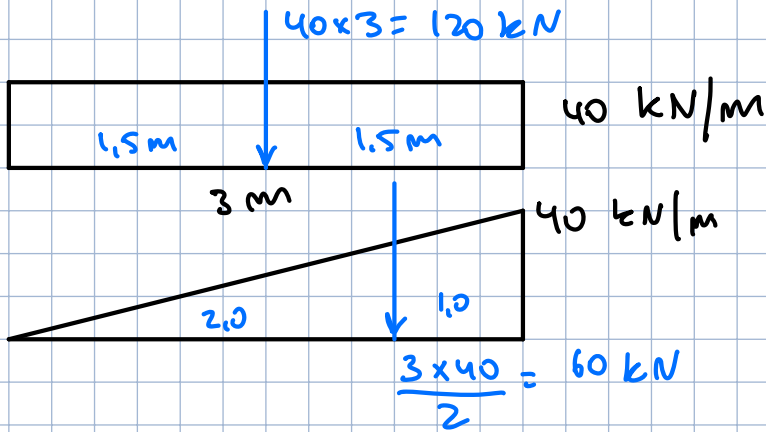
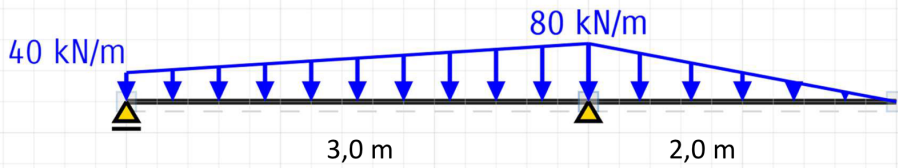
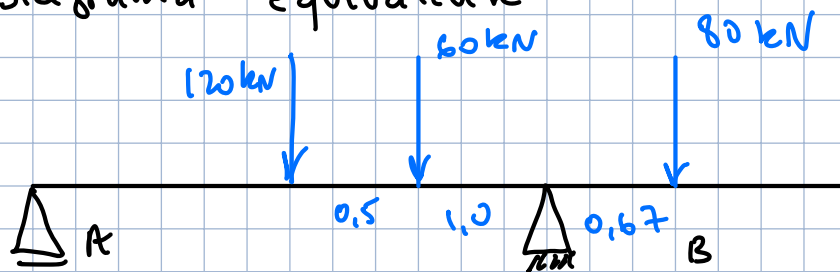


Diagrama equivalente



$$\sum F_v = R_A - 120 - 60 + R_B - 80 = 0 \Rightarrow R_A + R_B = 260$$

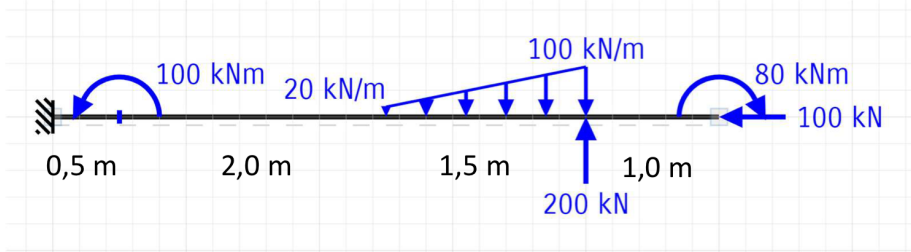
$$\sum M = R_A \cdot 0 - 120 \cdot 1.5 - 60 \cdot 2 + R_B \cdot 3 - 80 \cdot 3.67 = 0$$

$$R_B = 197.7 \text{ kN}$$

$$R_A = 260 - 197.7 = 62.1 \text{ kN}$$

$$R_A = 62 \text{ kN} ; \quad R_B = 198 \text{ kN}$$

3



$$\frac{1.5 \cdot 80}{2} = 60 \text{ kN}$$

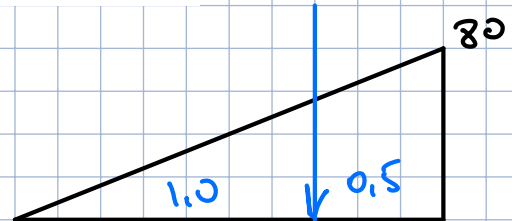
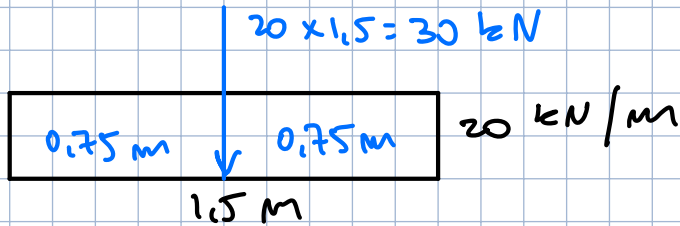
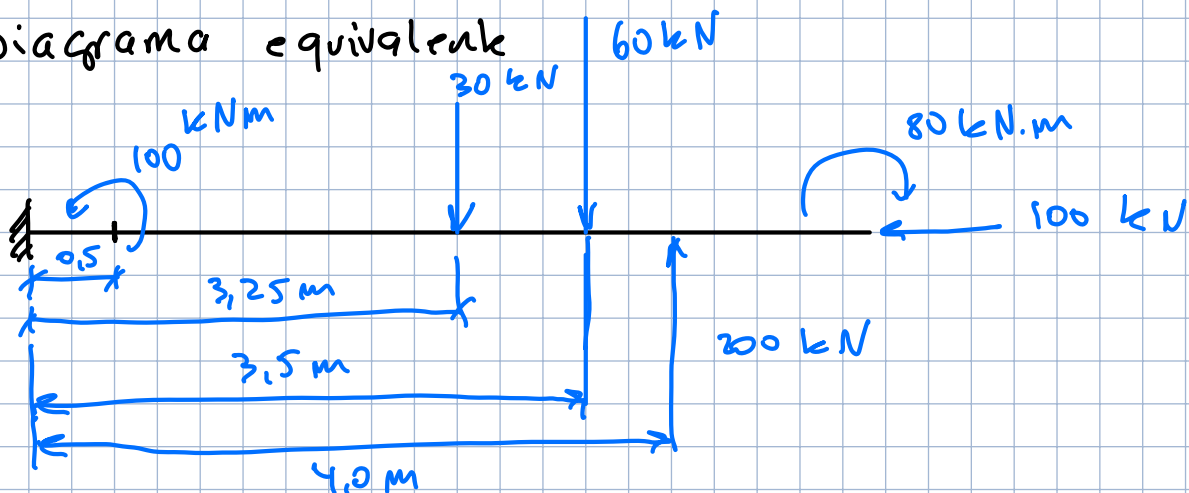


Diagrama equivalente



$$\sum F_v = R_y - 30 - 60 + 200 = 0 \quad R_y = -110 \text{ kN}$$

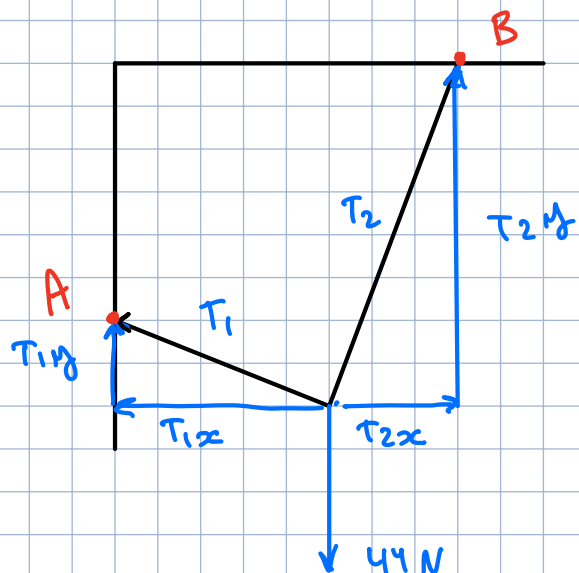
$$\sum F_H = 0 \quad R_x - 100 = 0 \quad R_x = 100 \text{ kN}$$

$$\sum M = 0 \quad M + 100 - 30 \cdot 3.25 - 60 \cdot 3.5 + 200 \cdot 4 - 80 = 0$$

$$M = -512.5 \text{ kN} \cdot \text{m}$$

$$R_x = 100 \text{ kN} \rightarrow \quad R_y = -110 \text{ kN} \downarrow \quad M = -512 \text{ kN} \cdot \text{m} \curvearrowright$$

calcule a reação nos apoios A e B



$$\sum F_H = 0 \Rightarrow -T_1x + T_2x = 0$$

$$T_1x = T_2x$$

$$\sum F_V = 0 \quad T_1y + T_2y - 44 = 0$$

$$T_1y + T_2y = 44$$

$$\begin{cases} 0,87 \cdot T_1 = 0,5 \cdot T_2 & \Rightarrow T_2 = 1,74 T_1 \\ 0,5 T_1 + 0,87 T_2 = 44 \end{cases}$$

$$0,5 T_1 + 0,87 \cdot 1,74 \cdot T_1 = 44$$

$$2,01 T_1 = 44$$

$$T_1 = 21,8 \Rightarrow T_1 = 22 \text{ N}$$

$$T_2 = 1,74 \cdot T_1 \Rightarrow T_2 = 38 \text{ N}$$

